The Impact of Student Grade-Level on the Honors Algebra I Midterm Exam Results

Technical Report

Clint Sattler
Supervisor, Research and Evaluation
Knox County Schools
Department of Research, Evaluation, and Assessment

January 2018

Overview

The number of 7th grade students enrolled in Honors Algebra I increased dramatically during the 2017-2018 school year (SY1718) in the Knox County Schools (KCS). Twenty-five 7th grade students were enrolled in Honors Algebra I in SY1617 and sixty-three 7th grade students were enrolled in SY1718.

The available literature offers diverging views regarding the benefits of accelerating students in their mathematical instruction. Some studies have documented positive effects when exposing students to rigorous Algebra instruction regardless of prior math performance and student demographics (Gamoran, 2000, Domina, 2014). Similar findings have spurred states and districts to champion "Algebra for all" initiatives, but the results of these initiatives have been somewhat mixed (Nomi, 2012). Some research also suggests that acceleration of math instruction is correlated to negative effects in the near-term (Clofelter, 2015, Marsh, 2016). These negative effects are possibly related to "gaps" in student knowledge (due to missing instruction in grade-level curriculum) and these "gaps" can compound as students progress to more advanced mathematical content (Loveless, 2008).

The KCS Mathematics Supervisor asked the Department of Research, Evaluation and Assessment to analyze the impact student grade-level may have on Algebra I outcomes. Results from the SY1718 (district created) Honors Algebra I midterm exam were chosen as the data source for this preliminary study. Using the midterm exam results allows KCS to understand the dynamics of student learning prior to the release of any state test results.

Methodology

Hierarchical Linear Modeling (HLM) was used to model student performance on the Honors Algebra I midterm exam because of the nested structure of the data. The student-level data was nested under two entities: teacher and school. A three-level HLM with random intercepts at both the school and teacher levels was used to model the data. The analysis included two independent variables at the student level. No group-level independent variables were included at the teacher and school levels.

One of the HLM independent variables was a measure of student academic ability. Projected state percentiles for the Algebra I End-of-Course (EOC) state exam were chosen as the most appropriate measures of student ability. These values were available via the Tennessee Value Added Assessment System (TVAAS) and were derived from past performance on state assessments. Projected state percentiles were converted to normal curve equivalents (NCEs) and squared. The inverse of the resulting squared NCEs were normalized in an effort to linearize the data to the maximum extent possible.

The other independent variable included in the HLM model was the students' grade-level. Student grade-levels were entered as factors into the HLM model with 8th grade set as the reference level. The dependent variable in the HLM regression was the normalized percent of correct responses on the Honors Algebra I midterm exam.

Honors Algebra I midterm exam results were available for 1,096 KCS middle school students. Sixteen student records were removed from the sample due to missing Algebra I TVAAS projections, leaving 1,080 records for statistical modeling. HLM was completed in R version 3.4.3 and RStudio version 1.0.143 using the lme4 package.

Results

The raw statistics from the Honors Algebra I midterm are contained in Table 1.

Table 1: Honors Algebra I Midterm Statistics

Mean % Correct	Standard Deviation	1st Quartile	Median	3rd Quartile	Minimum Score	Maximum Score
81.5%	15.4%	75.0%	85.0%	95.0%	15.0%	100%

The HLM model used to fit the data was as follows:

$$y_{ijk} = \pi_{0jk} + \pi_1 * Prev.Ach + \pi_2 * Grade\ Level + \epsilon_{ijk}$$

$$\pi_{0jk} = \beta_{00k} + r_{0jk}$$

$$\beta_{00k} = \gamma_{000} + \mu_{00k}$$

The prediction model appears to fit the observed data satisfactorily (Figure 1). Readers should note that there was a significant portion of variance in student performance on the Honors Algebra I midterm exam that was not described by the model.

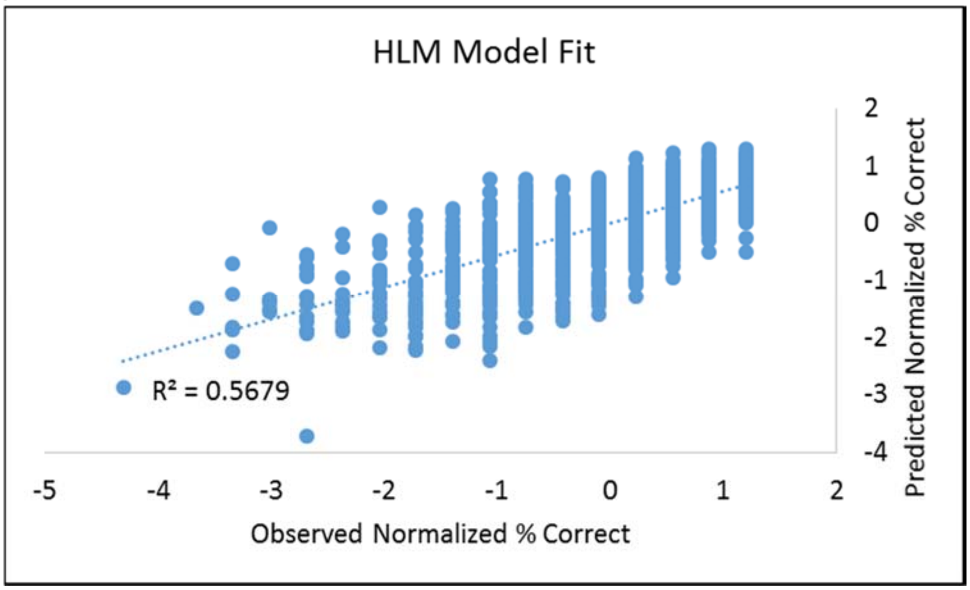

Figure 1: Observed Vs. Predicted (HLM) Values

Residual analysis indicates that the fit to the model may contain some bias because of non-linearies in the data. This is most likely due to low outliers and ceiling effects (no student could score higher than 100%). The histogram and Q-Q plot of the residuals are contained in Figures 2 and 3 respectively. Efforts were made to transform the raw data (using inverse, square root, and logarithmic transformations) to obtain a better fit but without success. Winnowing of the sample was not attempted since there was no empirical or theoretical reason to remove outlier data.

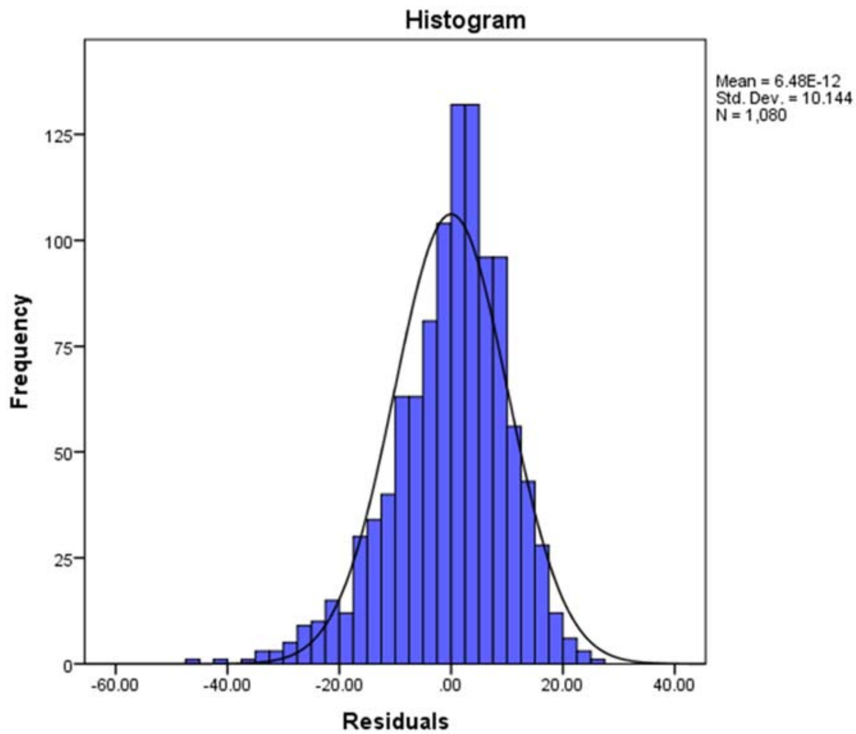

Figure 2: Histogram of HLM Residuals

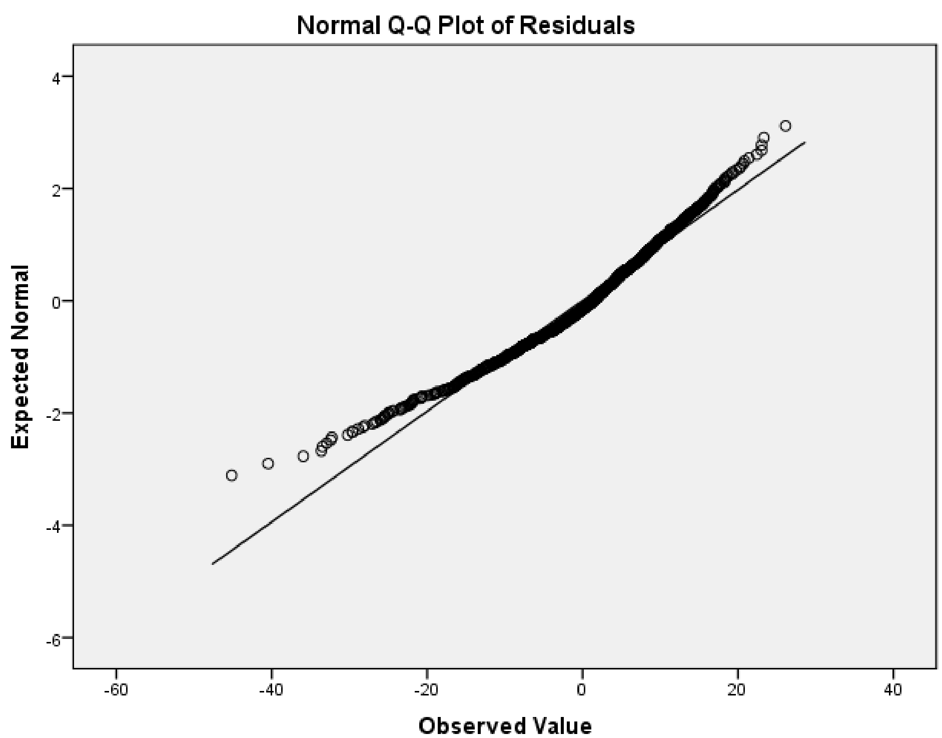

Figure 3: Q-Q Plot of HLM Residuals

The parameter estimates and the statistics associated with the fixed effects are available in Table 2. Random effects from the HLM regression (and their associated standard errors) are available in the appendix.

Table 2: HLM Fixed Effects

Parameter	Estimate	Std. Error	t
Intercept	-0.16	0.15	-1.02
Projected State NCE ($1/NCE^2$, normalized)	-0.37	0.02	-15.13
Grade = 7	-0.23	0.11	-2.21
Grade = 6	-0.09	0.30	-0.30

We can reject the null hypothesis that the results of the Honors Algebra I midterm were no different for 7th grade students when compared to 8th grade students when all other factors were held constant ($p=0.027$, $\alpha=0.05$). The magnitude of the difference was estimated at -0.23 standard deviations, which correlated to 3.6% of the points available on the test. In practical terms, an Honors Algebra I student who was in 7th grade would be expected to miss approximately ¾ of a question more than an 8th grade student with a similar academic history and the same teacher/school assignment. When the standard error of the estimate is considered, a 7th grade student would be expected to miss anywhere between ¼ and 1 ¼ more questions than an 8th grade student with similar academic history and the same teacher/school assignment.

Conclusions & Considerations

After controlling for previous academic performance, teacher assignment and school assignment, there was evidence that 8th grade students were likely to outperform 7th grade students on the Honors Algebra I midterm exam. It is estimated that 7th grade students missed between ¼ and 1 ¼ questions more than 8th grade students with similar academic history and the same teacher/school assignment. Although this finding is statistically significant, it may not be practically significant due to the relatively small magnitude of the difference.

Future studies should determine if similar results are detectable on external summative exams. The Knox County Schools should try to understand the impact of missing direct instruction of the 7th and 8th grade math curriculum on results in state or national exams relating to Algebra I, Geometry, and more advanced math topics.

References

Clotfelter, C., Ladd, H., & Vigdor, J., (2015). The Aftermath of Accelerating Algebra: Evidence from District Policy. Journal of Human Resources, 50(1), 159-188.

Domina, T., (2014). The Link between Middle School Mathematics Course Placement and Achievement. Child Development. 85, 1948-1964.

Gamoran, A. & Hannigan, E., (2000). Algebra for Everyone? Benefits of College-Preparatory Mathematics for Students with Diverse Abilities in Early Secondary School. Educational Evaluation and Policy Analysis, 22(3), 241-254.

Loveless, T., (2008). The Misplaced Math Student: Lost in Eighth-Grade Algebra. The 2008 Brown Center Report on American Education. Special Release. Washington D.C.: Brookings.

Marsh, H. W. (2016). Cross-cultural generalizability of year in school effects: Negative effects of acceleration and positive effects of retention on academic self-concept. Journal of Educational Psychology, 108(2), 256-273.

Nomi, T., (2012). The Unintended Consequences of an Algebra-for-All Policy on High-Skill Students: Effects on Instructional Organization and Students' Academic Outcomes. Educational Evaluation and Policy Analysis, 34(4), 489-505.

SAS Institute Inc., (2017). SAS EVAAS Technical Documentation of 2017 TVAAS Analyses. Cary, NC: SAS Institute Inc.

Appendix

Table 3: HLM Random Effects

Level	Teacher	Random Effect	Level	School	Random Effect
r_{0jk}	Teacher A	0.096	μ_{00k}	Bearden Middle School	0.114
r_{0jk}	Teacher B	0.121	μ_{00k}	Carter Middle School	0.086
r_{0jk}	Teacher C	-0.223	μ_{00k}	Cedar Bluff Middle School	-0.066
r_{0jk}	Teacher D	-0.031	μ_{00k}	Farragut Middle School	0.510
r_{0jk}	Teacher E	-0.337	μ_{00k}	Gresham Middle School	0.217
r_{0jk}	Teacher F	0.239	μ_{00k}	Halls Middle School	0.432
r_{0jk}	Teacher G	-0.409	μ_{00k}	Holston Middle School	-0.606
r_{0jk}	Teacher H	-0.031	μ_{00k}	Karns Middle School	0.323
r_{0jk}	Teacher I	-0.206	μ_{00k}	Northwest Middle School	-0.735
r_{0jk}	Teacher J	0.085	μ_{00k}	Powell Middle School	-0.001
r_{0jk}	Teacher K	0.108	μ_{00k}	South Doyle Middle School	0.225
r_{0jk}	Teacher L	0.000	μ_{00k}	Vine Middle Magnet	-0.796
r_{0jk}	Teacher M	0.032	μ_{00k}	West Valley Middle School	0.698
r_{0jk}	Teacher N	0.000	μ_{00k}	Whittle Springs Middle	-0.402
r_{0jk}	Teacher O	-0.113			
r_{0jk}	Teacher P	0.063			
r_{0jk}	Teacher Q	0.361			
r_{0jk}	Teacher R	0.285			
r_{0jk}	Teacher S	-0.019			
r_{0jk}	Teacher T	0.036			
r_{0jk}	Teacher U	-0.253			
r_{0jk}	Teacher V	-0.061			
r_{0jk}	Teacher W	0.195			
r_{0jk}	Teacher X	0.061			

Table 4: Standard Errors of Random Effects

Level	Teacher	Std Error	Level	School	Std Error
r_{0jk}	Teacher A	0.17	μ_{00k}	Bearden Middle School	0.19
r_{0jk}	Teacher B	0.24	μ_{00k}	Carter Middle School	0.20
r_{0jk}	Teacher C	0.25	μ_{00k}	Cedar Bluff Middle School	0.25
r_{0jk}	Teacher D	0.17	μ_{00k}	Farragut Middle School	0.16
r_{0jk}	Teacher E	0.20	μ_{00k}	Gresham Middle School	0.20
r_{0jk}	Teacher F	0.19	μ_{00k}	Halls Middle School	0.25
r_{0jk}	Teacher G	0.19	μ_{00k}	Holston Middle School	0.19
r_{0jk}	Teacher H	0.17	μ_{00k}	Karns Middle School	0.16
r_{0jk}	Teacher I	0.24	μ_{00k}	Northwest Middle School	0.26
r_{0jk}	Teacher J	0.18	μ_{00k}	Powell Middle School	0.25
r_{0jk}	Teacher K	0.17	μ_{00k}	South Doyle Middle School	0.26
r_{0jk}	Teacher L	0.20	μ_{00k}	Vine Middle Magnet	0.27
r_{0jk}	Teacher M	0.17	μ_{00k}	West Valley Middle School	0.16
r_{0jk}	Teacher N	0.24	μ_{00k}	Whittle Springs Middle	0.27
r_{0jk}	Teacher O	0.25			
r_{0jk}	Teacher P	0.25			
r_{0jk}	Teacher Q	0.20			
r_{0jk}	Teacher R	0.19			
r_{0jk}	Teacher S	0.24			
r_{0jk}	Teacher T	0.17			
r_{0jk}	Teacher U	0.19			
r_{0jk}	Teacher V	0.17			
r_{0jk}	Teacher W	0.18			
r_{0jk}	Teacher X	0.20			